 Normat **57**:2, 96 (2009)

Summary in English

David Cox, *Why Eisenstein proved the Eisenstein criterion and why Schönemann discovered it first.*
The Eisenstein criterion is well-known to all students of algebra, and it gives a very simple proof of the irreducibility of the cyclotomic polynomial $\Theta_p(x) = x^{p-1} + x^{p-2} + \cdots + 1$ for p prime. The irreducibility was (of course) known to Gauss but via a much more complicated proof. Simple ideas almost always arrive through tortuous detours, and the Eisenstein criterion being no exception, was originally stated for Gaussian integers in connection with polynomials associated to the division problem on the lemniscate! When it was published in Crelle in 1850, it provoked a complaint from a now forgotten mathematician Schönemann (1812-68) who had published a different proof of what was essentially the same criterion in the same journal only a few years earlier. His formulation is maybe even more elegant. Assume that $f(x) \in \mathbb{Z}[x]$ is of degree $n > 0$ and that there is some prime p and an integer a such that $f(x) = (x-a)^n + pF(x)$. If $F(a) \neq 0(p)$ then $f(x)$ irreducible mod p^2. This can be applied to the well known fact that $x^p - 1 = (x-1)^p(p)$ for p prime to get the irreducibility of $\Theta_p(x)$.

Schönemann not only deserves the priority for the Eisenstein criterion (and for some time his name was actually attached to Eisenstein's) but he also anticipated Hensel's lemma (which, however, in some weaker form was later found in the *Nachlaß* of Gauss), and did work out a theory for finite fields, although of course scooped by both Gauss and Galois. Yet the modern presentation of the latter subject did not appear until the very end of the 19th century when lectured on by E.H. Moore.

In addition to the main story a survey of the remarkable achievements of Gauss and Abel on elliptic functions is presented as providing the backdrop to the work of Schönemann and Eisenstein. Incidentally, the modern notion of an Abelian group, abstracted from the complicated equations on division points studied by Abel, did not appear until 1896 in Weber's classical *Lehrbuch der Algebra*.

Lars Gårding, *Näringskedjor* (Swedish).
A food-chain is an ordered collection of populations feeding on each other. A simple linear model for the variations of the populations is introduced. A condition for stability, i.e. that the variations of the populations are bounded, is that all the eigenvalues of the corresponding matrices lie on the unit-circle. The populations will then vary cyclically. The condition gives some restrictions on the fertility and life-expectancies of the various populations. Special cases of simple chains are studied and compared with real-life data, with some remarkable good fits.

Trond Steihaug and D.G. Rogers, *A minimum requiring angle trisection.*
Given a piece of foldable material (e.g. paper) in the shape of a right-angled triangle. Fold it by placing the right-angled corner on the hypothenuse. How should this be done in order to minimize the area of the folded triangle? The problem leads to a cubic equation whose relevant solution turns out to involve an angle trisection (and thus obtainable by a succession of foldings). Variations of the problem are considered, in particular letting one of the acute corners instead be placed on the opposite side.