

Summary in English

Marius Overholt, *Summer av to kvadrat* (Norwegian).

The characterization of numbers representable as the sum of two squares in terms of their divisors has been known since the 17th century (Girard, Fermat) but not proved until 18th century (Euler). A refinement involving the number $R(n)$ of representations of n as sum of two squares was proved by Jacobi using theta functions. This article starts out by reproducing a very elementary proof due to Heath-Brown of Fermat's characterization of primes of type $4n + 1$. Then it turns to discussing problems concerning asymptotic distribution, occurrence in prescribed intervals, infinite occurrences of patterns such as $n, n + h_1, n + h_2 \dots h_k$ for fixed h_i (It is shown that $n, n + 1, n + 2$ occurs infinitely often, but of course $n, n + 1, n + 2, n + 3$ may never all be sums of two squares.) Those are compared with the corresponding statements for primes. One may note that the asymptotic behaviour of $B(x) = \sum_{n=a^2+b^2 < x} 1$ is more difficult to prove (Landau) than the prime number theorem, but the corresponding problem of $R(x) = \sum_{n=a^2+b^2 < x} R(n)$ taking into account the number $R(n)$ of representations of n is much easier to handle and can naturally be linked with elementary asymptotic formulas for the number of lattice points inside circles as observed by Gauss. Finally one may expect that every interval $[x, x + h]$ contains a sum of two squares if $h > C \log x$ for some suitable constant C , but so far it has only been shown for $h > Cx^{\frac{1}{4}}$. But if we relax the condition to almost all such intervals, there is a complete solution.

O. Znamenskaya & A. Shchuplev, *Real Toric Surfaces*.

In this note is proved that every real toric smooth orientable surface is topologically a torus. The presentation is self-contained and uses no prior knowledge of toric varieties.

A. Percy and D. G. Rogers, *Alternative route: from van Schooten to Ptolemy*.

A cyclic quadrilateral is a polygon with four vertices, all of which lie on a circle. Such enjoy some special properties. It is well-known that the sum of opposite angles always add up to π , it is maybe less well-known that the rectangle formed by the diagonals has the same area as the sum of the rectangles made up by opposite sides. The latter is known as Ptolemy's theorem. It has many consequences, not only of trigonometric computations, but also of justifying the elegant solution of the Dutch mathematician van Schooten (1615-60) of constructing the minimal sum of distances from a point to the vertices of a triangle, a problem posed as a challenge by Fermat. The article gives a historical survey and indicates how van Schooten's construction could serve as an inspiration by 'cutting and pasting' to suggest and prove Ptolemy's theorem.

Ulf Persson, *Trianglar med givna omskrivna och inskrivna cirklar*. (Swedish)

This is a second article in a series of three, concerning 1-dimensional families of triangles. In this case those with given circumscribed and inscribed radii - R, r are presented. The construction of such families is a special case of Poncelet's theorem, exploiting the approach via elliptic curves, initiated by Jacobi.