Summary in English

Christer O. Kiselman, *Euclid's straight lines* (English)

What is a straight line? Primitive notions are of course notoriously intractable as to formal definitions, but need to be understood through the way they are actually used. The author takes as point of departure Euclid's propositions 27 and 16 in his first book. The first gives a sufficient criterion for two lines to be parallel, the second states that the exterior angle is larger than any of the two opposite angles in a triangle. From a strict logical point of view the propositions do not follow from the axioms, as one can give a model (the projective plane) for which they do not hold. Clearly Euclid made some implicit assumptions. As Hilbert et al. pointed out about a century ago, Euclid made many implicit assumptions, which do not, however, detract from his achievement, so the focus of interest is to try and pinpoint more exactly how Euclid really thought (as opposed to what he wrote down). This leads the author on an historical and linguistic odyssey, with special emphasis on the notion of 'eutheia' which can be understood as meaning a line segment, a ray, or a line indefinitely extended, and used in all three meanings by Euclid. The notion of infinite extension leads to philosophical questions about potential versus actual infinity, and how we in retrospect can through the notion of equivalence classes speak about the infinite line without specifying it as a concrete geometric object unlike the finite line segment. Another issue discussed, if briefly, is the legitimacy of relying on visual diagrams in formal deductive proofs. To use them as support, be it for the imagination or memory, is one thing, but to draw actual conclusions from them, quite another thing. Figures can be misleading especially when one draws planar diagrams of geometrical configurations on a sphere not to mention a non-orientable surface. Was Euclid aware of the latter possibility? According to the author we can only speculate.

Leif Önneflod, *Comparison and Change* (English).

There is a discrepancy between mathematical symbolism and what mathematical concepts really mean. The former is a poor substitute for the latter. The problem is most acute in elementary instruction concerning the basic operations of arithmetic, leading to much confusion among pupils. Misconceptions may never be clarified, thus sustaining into adulthood, becoming a potential obstruction for using mathematics in everyday situations. The article can be seen as an introduction to a longer opus by the author in which he gives a detailed and systematic explanation of what it really means to perform elementary arithmetic operations with special emphasis on the didactic confusions involved.

The former is a poor substitute for the latter. The problem is most acute in elementary instruction concerning the basic operations of arithmetic, leading to much confusion among pupils (and educators). Misconceptions may never be clarified, thus sustaining into adulthood , becoming a potential obstruction for using mathematics in everyday situations. The article can be seen as an introduction to a longer opus by the author in which he gives a detailed and systematic explanation of what it really means to perform elementary arithmetic operations with special emphasis on the didactic confusions involved.